Last Update: September 16, 2025

NSFC–JSPS Joint Conference

# Current Status and New Development in the Theoretical Analysis for the Discrete Models of Partial Differential Equations

(CJ Numerical PDEs 2025)

## Date:

September 21–25, 2025

- Sunday, September 21, is for registration
  (and for the organizational committee preparatory meeting)
- Monday, September 22—Wednesday, September 24 are academic sessions
- Thursday, September 25, is for free discussion

## Venue:

Institute of Fundamental and Frontier Sciences (IFFS)
University of Electronic Science and Technology of China, Chengdu

## Organizers

- Guanyu Zhou, University of Electronic Science and Technology of China
- Norikazu Saito, The University of Tokyo

## Sponsors

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- Grant-in-Aid for Scientific Research (A) (JSPS KAKENHI Grant Number JP21H04431)

## Timetable

| Day 1: Monday | Day 2: Tuesday | Day 3: Wednesday |
|---|---|---|
| 8:45-10:30 Session 1<br>Chair: N. Saito<br>8:45-9:00 Opening<br>9:00-9:30 E. Chung<br>9:30-10:00 T. Matsuo<br>10:00-10:30 Y. Yang | 8:40-10:40 Session 5<br>Chair: D. Furihata<br>8:40-9:10 H. Li<br>9:10-9:40 Y. Nakano<br>9:40-10:10 Y. Gu<br>10:10-10:40 Y. Miyatake | 8:40-10:40 Session 7<br>Chair: K. Kobayashi<br>8:40-9:10 J. Wang<br>9:10-9:40 X. Liu<br>9:40-10:10 T. Yin |
| 10:30-11:00<br>Break & Poster Session 1<br>J. Hu, Y. Taniguchi | 10:40-11:10<br>Break & Poster Session 3<br>Y. Zhou, K. Matsui | 10:10-10:40<br>Break & Poster Session 4<br>Y. Chiba, C. Wang |
| 11:00-12:30 Session 2<br>Chair: E. Chung<br>11:00-11:30 T. Tsuchiya<br>11:30-12:00 H. Shen<br>12:00-12:30 D. Furihata | 11:10-12:30<br>Session 6 (Plenary Lectures)<br>Chair: L. Xu<br>11:10-11:50 W. Zheng<br>11:50-12:30 N. Saito | 10:40-12:10 Session 8<br>Chair: J. Wang<br>10:40-11:10 Y. Sugitani<br>11:10-11:40 W. Lei<br>11:40-12:10 S. Uchiumi |
| 12:30-14:30 Lunch | 12:30-14:30 Lunch | 12:10-14:30 Lunch |
| 14:30-16:00 Session 3<br>Chair: T. Tsuchiya<br>14:30-15:00 J. Hu<br>15:00-15:30 H. Suito<br>15:30-16:00 C. Wei | Free Discussion | 14:30-16:00 Session 9<br>Chair: T. Matsuo<br>14:30-15:00 K. Wang<br>15:00-15:30 S. Sato<br>15:30-16:00 F. Jing |
| 16:00-16:30<br>Break & Poster Session 2<br>Y. Ren, T. Matsumoto | Free Discussion | 16:00-16:30<br>Break & Poster Session 5<br>H. Takemura, H. Nakamura |
| 16:30-18:00 Session 4<br>Chair: C. Wei<br>16:30-17:00 T. Ide<br>17:00-17:30 M. Lyu<br>17:30-18:00 K. Kobayashi | Free Discussion | 16:30-18:00 Session 10<br>Chair: G. Zhou<br>16:30-17:00 M. Okumura<br>17:00-17:30 X. Hu<br>17:30-18:00 Y. Ueda |

# Program

## Day 1: Monday, September 22

### Session 1 (Chair: N. Saito)

- 8:45-9:00 Opening
- 9:00-9:30 Eric Chung (The Chinese University of Hong Kong)

  Robust Computational Methods for Helmholtz equations in high contrast heterogeneous media
- 9:30-10:00 Takayasu Matsuo (The University of Tokyo)

  A new convergence estimate of an energy-preserving scheme for the KdV equation
- 10:00-10:30 Yin Yang (Xiangtan University)

  Multiscale Model Reduction in Heterogeneous Perforated Domains Based on CEM-GMsFEM

### Break & Poster Session 1

- 10:30-11:00 Jingyan Hu (University of Electronic Science and Technology of China)

  A projection approach to the Peterlin viscoelastic model with the continuous interior penalty finite element method
- 10:30-11:00 Yasutoshi Taniguchi (The University of Tokyo)

  Dirichlet boundary stabilization methods for space–time finite element analysis: An application to the wave equation

### Session 2 (Chair: E. Chung)

- 11:00-11:30 Takuya Tsuchiya (The University of Osaka)

  Recent development of the finite element error analysis on anisotropic meshes
- 11:30-12:00 Hua Shen (University of Electronic Science and Technology of China)

  Arbitrarily High-Order Compact Central Schemes for Solving Conservation Laws
- 12:00-12:30 Daisuke Furihata (The University of Osaka)

  A hybrid numerical scheme combining finite volume and particle methods for partial differential equations

**Session 3 (Chair: T. Tsuchiya)**

- 14:30-15:00 Jiashun Hu (Hong Kong Polytechnic University)

  Energy dissipating ALE-MDR method for Navier–Stokes free boundary problems with moving contact line

- 15:00-15:30 Hiroshi Suito (Tohoku University)

  On the thermal convection in intraocular aqueous humor flows

- 15:30-16:00 Chaozhen Wei (University of Electronic Science and Technology of China)

  Efficient operator splitting methods for minimizing movement schemes of Wasserstein-like gradient flows

**Break & Poster Session 2**

- 16:00-16:30 Yicheng Ren (University of Electronic Science and Technology of China)

  An energy-preserving numerical approach based on Helmholtz decomposition for the Maxwell-Schrödinger system

- 16:00-16:30 Takehiro Matsumoto (Tohoku University)

  Finite element approaches to the thermal convection in the eye

**Session 4 (Chair: C. Wei)**

- 16:30-17:00 Takanori Ide (Josai University)

  Reconstruction of the inclusions in electrical impedance tomography using enclosure neural networks for experimental data

- 17:00-17:30 Maohui Lyu (Beijing University of Posts and Telecommunications)

  Nodal discontinuous Galerkin methods for Maxwell's equations in Lorentz-Kerr-Raman medium without nonlinear algebraic solver

- 17:30-18:00 Kenta Kobayashi (Hitotsubashi University)

  Error analysis of Lagrange interpolation on triangles and tetrahedra

## Day 2: Tuesday, September 23

**Session 5 (Chair: D. Furihata)**

- 8:40-9:10 Haigang Li (Beijing Normal University)

  Higher derivative estimates and numerical simulations for Lamé system with closely spaced rigid inclusions

- 9:10-9:40 Yumiharu Nakano (Institute of Science Tokyo)

  Convergence of differentiable approximation schemes for fully nonlinear parabolic equations

- 9:40-10:10 Yiqi Gu (University of Electronic Science and Technology of China)

  Layer separation models with weighted auxiliary variables in deep learning

- 10:10-10:40 Yuto Miyatake (The University of Osaka)

  Statistical modelling for quantifying discretization errors

**Break & Poster Session 3**

- 10:40-11:10 Yuxuan Zhou (University of Electronic Science and Technology of China)

  Arbitrarily High-order Compact Central Schemes with Multi-resolution for Solving Hyperbolic Conservation Systems

- 10:40-11:10 Kazunori Matsui (Tokyo University of Marine Science and Technology)

  Numerical method for an elastoplastic model with time-dependent thresholds

**Session 6 (Plenary Lectures) (Chair: L. Xu)**

- 11:10-11:50 Weiying Zheng (Chinese Academy of Sciences)

  A new perfectly matched layer method for the Helmholtz equation in nonconvex domains

- 11:50-12:30 Norikazu Saito (The University of Tokyo)

  Decoupling iterative numerical methods for mean field games

## Day 3: Wednesday, September 24

**Session 7 (Chair: K. Kobayashi)**

- 8:40-9:10 Jilu Wang (Harbin Institute of Technology)

  Optimal $L^2$ error estimates of unconditionally stable FE schemes for the Cahn-Hilliard-Navier-Stokes system

- 9:10-9:40 Xuefeng Liu (Tokyo Woman's Christian University)

  Rigorous computation for eigenvalue problems and solution verification for the Navier–Stokes equations

- 9:40-10:10 Tao Yin (Academy of Mathematics and Systems Science)

  Adaptive DtN-FEM for the thermo/poro-elastic wave scattering problems

**Break & Poster Session 4**

- 10:10-10:40 Yuki Chiba (The University of Tokyo)

  DG time-stepping method for abstract parabolic problem

- 10:10-10:40 Chang Wang (University of Electronic Science and Technology of China)

  Well-posedness of viscoelastic contact problems with modified Signorini, Tresca-friction, and Clarke-subdifferential type contact conditions incorporating both velocity and displacement

**Session 8 (Chair: J. Wang)**

- 10:40-11:10 Yoshiki Sugitani (Josai University)

  Numerical simulation and identification of time-varying parameters in the glucose–insulin metabolic system for ICU patients

- 11:10-11:40 Wenyu Lei (University of Electronic Science and Technology of China)

  Finite element approximations for elliptic problem with regularized data: error estimates and adaptivity

- 11:40-12:10 Shinya Uchiumi (Hokkaido University)

  A pressure-stabilized projection Lagrange–Galerkin scheme for the transient Oseen problem

**Session 9 (Chair: T. Matsuo)**

- 14:30-15:00 Kun Wang (Chongqing University)

  Structure preserving numerical schemes for solving the Keller-Segel chemotaxis equations

- 15:00-15:30 Shun Sato (Tokyo Metropolitan University)

  Linearly implicit dissipative integrators inspired by proximal DC algorithms from optimization

- 15:30-16:00 Feifei Jing (Northwestern Polytechnical University)

  Finite Volume Methods for Stokes Variational Inequalities

**Break & Poster Session 5**

- 16:00-16:30 Haruki Takemura (The University of Tokyo)

  Error estimates of semi-Lagrangian schemes using high-order interpolation for nonlinear advection–diffusion equations

- 16:00-16:30 Haruka Nakamura (The University of Tokyo)

  On the discretization and convergence rates of an iterative method for Mean Field Games

**Session 10 (Chair: G. Zhou)**

- 16:30-17:00 Makoto Okumura (Konan University)

  Recent progress in the structure-preserving schemes for the two-dimensional Cahn-Hilliard models with dynamic boundary conditions

- 17:00-17:30 Xin Hu (Wuhan University)

  The incompressible Navier-Stokes limit from the lattice BGK Boltzmann equation

- 17:30-18:00 Yuki Ueda (Hokkaido University)

  Numerical computations of split Bregman method for fourth-order total variation flow

# Abstracts

## Chinese side

(The list is arranged in alphabetical order by surname.)

### Eric Chung (The Chinese University of Hong Kong)

Robust Computational Methods for Helmholtz equations in high contrast heterogeneous media

Solving Helmholtz equations with heterogeneous coefficients can be challenging due to the high-contrast structure and pollution effect. In this talk, we present a novel multiscale method in the spirit of the constraint energy minimization generalized multiscale finite element method (CEM-GMsFEM) by tailoring new trial and test spaces. We establish the inf-sup stability to secure the well-posedness of our multiscale problem and prove the error estimate that is independent of the high-contrast coefficient. The theoretical results are validated by numerical tests, which further show that the multiscale technique can effectively capture pertinent physical phenomena. This work is partially supported by the Hong Kong RGC General Research Fund (Projects: 14305423 and 14305222).

### Yiqi Gu (University of Electronic Science and Technology of China)

Layer separation models with weighted auxiliary variables in deep learning

We develop a new optimization framework for the least squares regression and partial differential equations via fully connected neural networks and physics-informed neural networks. The gradient descent method behaves inefficiently in deep learning due to the high non-convexity of loss functions and the vanishing gradient issue. Our work proposes layer separation models with self-adaptive weighted auxiliary variables. Specifically, we introduce auxiliary variables to separate the layers of the deep neural networks and construct self-adaptive weighted loss functions. We prove the consistency between the proposed weighted loss and the original mean squared loss. Numerical experiments are presented to verify the above theory and demonstrate the effectiveness and robustness of our models compared to gradient descent.

### Jiashun Hu (The Hong Kong Polytechnic University)

Energy dissipating ALE-MDR method for Navier–Stokes free boundary problems with moving contact line

Modeling incompressible Navier–Stokes flows with evolving free surfaces and moving contact lines poses significant challenges, both in capturing the correct physical energy dissipation and in maintaining mesh quality during long-time simulations. In this talk, I will present a new arbitrary Lagrangian–Eulerian finite element method that integrates two complementary strategies: the tangential motion approach of Barrett–Garcke–Nürnberg (BGN) on the free surface, and a minimal-deformation-rate (MDR) technique in the bulk and along solid boundaries. A key difficulty in three dimensions—the ambiguity of tangential motion at the moving contact line—is resolved by imposing a constraint that removes this instability. The resulting ALE–MDR method dissipates energy by design and preserves high-quality meshes even in demanding scenarios. Numerical experiments in both two and three dimensions will demonstrate the robustness and accuracy of the approach.

**Jingyan Hu (University of Electronic Science and Technology of China)**

A projection approach to the Peterlin viscoelastic model with the continuous interior penalty finite element method

We propose a projection method combined with the continuous interior penalty finite element method (CIP) to solve the Peterlin viscoelastic fluid model with a diffusion term. The projection step preserves the positive definiteness of the conformation tensor, while the CIP discretization ensures stability as the diffusion coefficient $\to 0$. A rigorous analysis of well-posedness and stability is provided. Several numerical experiments confirm the accuracy and effectiveness of the proposed projection scheme.

**Xin Hu (Wuhan University)**

The incompressible Navier-Stokes limit from the lattice BGK Boltzmann equation

We prove the incompressible Navier-Stokes (INS) limit for a discrete-velocity BGK Boltzmann equation (lattice BGK) in any spatial dimension $d \geq 2$. In the spirit of the Bardos-Golse-Levermore program, we derive uniform energy bounds for an approximate LBGK system and, using the $L^2$ Helmholtz projection and compactness, construct local weak solutions of INS as the Knudsen number $\epsilon \to 0$. For $d \in \{2, 3\}$, we obtain algebraic moment/isotropy conditions on finite velocity sets and probability weights that guarantee the INS limit. Two-dimensional computations indicate that, for the D2Q9 lattice, the gradient error decays as $O(\epsilon^2)$. These results give a rigorous hydrodynamic limit for lattice BGK models and clarify structural conditions under which discrete-velocity schemes recover incompressible fluid dynamics.

**Feifei Jing (Northwestern Polytechnical University)**

Finite Volume Methods for Stokes Variational Inequalities

The boundary condition of fluid-flow is one of the most important factors to determine its hydrodynamic behaviors. In microfluidic systems, boundary slip may have a significant effect on the performance of such system. As an effective technique to catch the boundary slip phenomenon, numerical method provides some theoretical guidance for related experimental research. In this talk, we analyze the lowest-order finite volume method for the Stokes and Navier-Stokes equations with a nonlinear slip boundary condition of friction type, which is used to describe the flow in the blood vessel of arteriosclerosis, as well as the possible slip phenomena. Due to the subdifferentiability of such boundary condition, these models can be characterized by variational or hemivariational inequalities. We will design some stable and efficient finite volume schemes, and establish priori error analyses for such variational inequalities. Numerical tests are reported to verify the theoretical results.

**Wenyu Lei (University of Electronic Science and Technology of China)**

Finite element approximations for elliptic problem with regularized data: error estimates and adaptivity

Approximations of the Dirac delta distribution are commonly used to create sequences of smooth functions approximating nonsmooth (generalized) functions, via convolution. In this talk we show a priori rates of convergence of this approximation process in standard Sobolev norms, with minimal regularity assumptions on the approximation of the Dirac delta distribution. The application of these estimates to the numerical solution of elliptic problems with singularly supported forcing terms allows us to provide sharp $H^1$ and $L^2$ error estimates for the corresponding regularized problem. As an application, we show how finite element approximations of a regularized immersed interface method results in the same rates of convergence of its non-regularized counterpart, provided that the support of the Dirac delta approximation is set to a multiple of the mesh size, at a fraction of the implementation complexity. Next, we provide an adaptive finite element algorithm to improve the convergence rate with respect to numbers of degrees of freedom. We show that the energy error decay is quasi-optimal in two-dimensional space and sub-optimal in three-dimensional space. Numerical simulations are provided to confirm our findings.

**Haigang Li (Beijing Normal University)**

Higher derivative estimates and numerical simulations for Lamé system with closely spaced rigid inclusions

In this talk we study the interaction between two closely spaced rigid inclusions embedded in an elastic material. It is well known that the stress significantly amplifies in the narrow region between the inclusions as the distance between them approaches zero. To effectively analyze the singular behavior of solutions, as well as to develop accurate numerical schemes, it is crucial to obtain higher-order derivative estimates—both from an engineering perspective and for the requirements of numerical experiments. We derive high-order derivative estimates and numerical scheme for the Lamé system in the presence of two rigid inclusions.

**Maohui Lyu (Beijing University of Posts and Telecommunications)**

Nodal discontinuous Galerkin methods for Maxwell's equations in Lorentz-Kerr-Raman medium without nonlinear algebraic solver

The propagation of electromagnetic waves is modeled by time-dependent Maxwell's equations coupled with constitutive laws that describe the responses of the media. In this work, we consider a nonlinear model that describes the electromagnetic wave in an optical medium with the linear Lorentz effect and the cubic nonlinear instantaneous Kerr and delayed Raman effects. Mathematically this model obeys an energy conservative/dissipative law. Though there have been active efforts in designing numerical methods to simulate this model, the methods proposed here are distinctive in that they are free of any nonlinear algebraic solvers. Moreover, in the absence of the Raman effect, our methods also enjoy a provable discrete energy law, and optimal a priori error estimates are further established when the exact solutions are sufficiently smooth. The key ingredients of the new methods include some novel treatment in time discretizations and nodal discontinuous Galerkin spatial discretization for the specific nonlinearities, and they also render a local nature of the methods and hence their suitability for parallel implementation with great efficiency. Numerical experiments are performed to illustrate the accuracy, stability, computational efficiency and parallel scalability of the proposed methods. We further apply the methods to simulate some physically relevant problems in one, two, and three dimensions.

**Yicheng Ren (University of Electronic Science and Technology of China)**

An energy-preserving numerical approach based on Helmholtz decomposition for the Maxwell–Schrödinger system

In this work, the numerical problem of the Maxwell-Schrödinger system under different gauges are concerned. We apply Helmholtz decomposition on the Maxwell-Schrödinger system under Lorentz, Coulomb and temporal gauges before using finite element approximation on it. Under this method, we can decompose the vector potential and estimate them on $\mathbf{H^1}$ rather than only on $\mathbf{H}(\mathbf{curl})$. Total charge and energy of the system are concerved in the finite element scheme. Convergence rates of different gauges and external potential fields are shown.

**Hua Shen (University of Electronic Science and Technology of China)**

Arbitrarily High-Order Compact Central Schemes for Solving Conservation Laws

In this talk, we will introduce a class of compact central schemes for solving conservation laws with arbitrarily high order. In this method, the solution is approximated by a piece-wise Pth-order polynomial and all the DOFs are stored and updated separately. The cell average is updated by a Riemann-solver-free finite volume scheme based on space-time staggered meshes. The kth-order spatial derivatives are updated by a compact central difference of the $(k-1)$th-order spatial derivatives at cell vertices. All the required space-time information is calculated by the exact or approximated Cauchy–Kovalevsky procedure. By doing so, the schemes can achieve arbitrarily uniform space-time high-order on a compact stencil consisting of only neighboring cells with only one explicit time stage. In addition, an efficient multi-resolution limiter is designed to suppress numerical oscillations near discontinuities.

**Chang Wang (University of Electronic Science and Technology of China)**

Well-posedness of viscoelastic contact problems with modified Signorini, Tresca-friction, and Clarke-subdifferential type contact conditions incorporating both velocity and displacement

We propose three modified contact boundary conditions incorporating both the velocity and the displacement with a parameter $\delta$ for the viscoelastic problem. As $\delta$ approaches 0, these conditions formally reduce to the conventional Signorini, Tresca-friction, and Clarke-subdifferential type boundary conditions, respectively. Consequently, the modified conditions, as a generalization of the conventional ones, can be viewed as contact conditions in the displacement with a dynamic setting. We derive weak formulations for the viscoelastic contact model under three modified contact conditions and explore their well-posedness. Additionally, we provide bounds on the weak solutions with respect to the parameter $\delta$.

**Jilu Wang (Harbin Institute of Technology)**

Optimal $L^2$ error estimates of unconditionally stable FE schemes for the Cahn-Hilliard-Navier-Stokes system

The paper is concerned with the analysis of a popular convex-splitting finite element method for the Cahn-Hilliard-Navier-Stokes system, which has been widely used in practice. Since the method is based on a combined approximation to multiple variables involved in the system, the approximation to one of the variables may seriously affect the accuracy for others. Optimal-order error analysis for such combined approximations is challenging. The previous works failed to present optimal error analysis in $L^2$-norm due to the weakness of the traditional approach. Here we first present an optimal error estimate in $L^2$-norm for the convex-splitting FEMs. We also show that optimal error estimates in the traditional (interpolation) sense may not always hold for all components in the coupled system due to the nature of the pollution/influence from lower-order approximations. Our analysis is based on two newly introduced elliptic quasi-projections and the superconvergence of negative norm estimates for the corresponding projection errors. Numerical examples are also presented to illustrate our theoretical results. More important is that our approach can be extended to many other FEMs and other strongly coupled phase field models to obtain optimal error estimates.

**Kun Wang (Chongqing University)**

Structure preserving numerical schemes for solving the Keller-Segel chemotaxis equations

In this talk, we consider two kinds of structure preserving numerical scheme for solving the Keller-Segel chemotaxis equations. First, based on the log-transformation, we study a first order fully discrete finite element scheme, which is proved to be mass conservation and positivity preserving. Its optimal error estimates are also deduced. Then, utilizing the Slotboom transformation and exponential time differencing (ETD) scheme, we propose parallelizable second order fully discrete finite difference scheme, which is proved to be mass conservation, positivity preserving, asymptotic behavior preserving and energy dissipation law preserving. Some numerical examples are shown to verify the correctness of the theoretical analyses.

**Chaozhen Wei (University of Electronic Science and Technology of China)**

Efficient operator splitting methods for minimizing movement schemes of Wasserstein-like gradient flows

In this talk, I will present a novel numerical approach based on minimizing movement schemes for a class of Wasserstein-like gradient flows arising widely in applications in material sciences such as phase separation, crystal growth, solid-state wetting/dewetting, thin film surfactant dynamics and reaction-diffusion dynamics. By leveraging the variational structure, along with the dynamical characterization of the Wasserstein-like transport distance, we construct a fully discrete scheme that constitutes a series of minimization problems with convex objective function and linear constraint. We construct a novel saddle-point formulation to address general nonlinear transport distances, and propose efficient operator splitting methods to solve the saddle-point problems. Our method has built-in positivity or bounds preserving, mass conservation, and entropy decreasing properties, and overcomes stability issue due to the strong nonlinearity and degeneracy. I will show a suite of simulation examples to demonstrate the effectiveness of our algorithm.

**Yin Yang (Xiangtan University)**

Multiscale Model Reduction in Heterogeneous Perforated Domains Based on CEM-GMsFEM

In this talk, we introduce a robust framework for tackling multiscale complexities in heterogeneous perforated domains using the Constraint Energy Minimizing Generalized Multiscale Finite Element Method (CEM-GMsFEM). Simulations in such domains are computationally challenging due to the varying scales of perforations and domain structures. Our method is applied to both the Poisson equation and linear elasticity problems within these domains. The framework consists of two main stages: the offline stage and the online stage. In the offline stage, we first solve an eigenvalue problem within each coarse block and then solve a minimization problem over an oversampled domain to construct multiscale basis functions. In the online stage, we generate online basis functions based on the local residual to rapidly reduce error. Additionally, we explore the impact of varying the number of oversampling layers on the construction of local basis functions. This approach provides an efficient and accurate solution framework for complex multiscale problems in perforated domains.

**Tao Yin (Academy of Mathematics and Systems Science)**

Adaptive DtN-FEM for the thermo/poro-elastic wave scattering problems

This talk will present our recent works on the numerical analysis of adaptive finite element method for solving the thermo/poro-elastic wave scattering problem in both two and three dimensions. The Dirichlet-to-Neumann (DtN) operator on a spherical surface is introduced via Helmholtz decomposition to reduce the scattering problem into a boundary value problem in a bounded domain. The well-posedness results are established for the variational problem and its modification due to the truncation of the DtN operator. A priori and a posteriori error estimates are developed, and the error estimates consist of both the finite element approximation error and the truncation error of the DtN operator. Numerical experiments are presented to illustrate the effectiveness of the proposed algorithm.

**Weiying Zheng (Academy of Mathematics and Systems Science)**

A New perfectly matched layer method for the Helmholtz equation in nonconvex domains

A new coupled perfectly matched layer (PML) method is proposed for the Helmholtz equation in the whole space with inhomogeneity concentrated on a nonconvex domain. Rigorous analysis is presented for the stability and convergence of the proposed coupled PML method, which shows that the PML solution converges to the solution of the original Helmholtz problem exponentially with respect to the product of the wave number and the width of the layer. An iterative algorithm and a continuous interior penalty finite element method (CIP-FEM) are also proposed for solving the system of equations associated to the coupled PML. Numerical experiments are presented to illustrate the convergence and performance of the proposed coupled PML method as well as the iterative algorithm and the CIP-FEM

**Yuxuan Zhou (University of Electronic Science and Technology of China)**

Arbitrarily High-order Compact Central Schemes with Multi-resolution for Solving Hyperbolic Conservation Systems

In this paper, we aim to construct a class of arbitrarily high-order compact central schemes with multi-resolution (AOCC-MR). The new schemes are developed based on high-order weighted compact central (WCC) schemes while retaining most advantages of WCC, namely possessing highly compact spatial stencils and achieving high-order time accuracy with one step via Cauchy-Kovalevski (C-K) procedure. Nevertheless, two primary challenges hinder the construction of high-order WCC schemes. Firstly, the derivation of high-order C-K procedures is relatively complex. Secondly, the CWENO-type limiter proposed in WCC is insufficient for suppressing the non-physical oscillations generated by high-order compact central schemes. To overcome these, we employ a Leibniz-rule-based algorithm to automate the C-K procedure, enabling a single program to implement arbitrarily high-order schemes, and design a problem-independent, unified-parameter MR limiter to suppress the non-physical oscillations across all scales.

## Japanese side

(The list is arranged in alphabetical order by surname.)

**Yuki Chiba (The University of Tokyo)**

DG time-stepping method for abstract parabolic promlem

DG time-stepping method is a numerical method for PDEs using piecewise continuous functions for time discretization. In the analysis on a smooth domain, it is used specific properties of domain approximation, which is abstracted as the relationship between a function space $V$ and finite-dimensional space $V_h$. We apply this to DG time-stepping method and get error estimate of DG time-stepping method for an abstract parabolic problem.

**Daisuke Furihata (The University of Osaka)**

A hybrid numerical scheme combining finite volume and particle methods for partial differential equations

To apply the particle method based on the continuity equation for nonlinear partial differential equations (conservative systems), which derives the law of conservation of mass, we developed a simple numerical scheme. This scheme defines density using Voronoi decomposition and discretizes it using the finite volume method concept. Additionally, to treat non-conservative systems, we introduced an artificial component that allows the particle mass to change over time, leading to the development of a hybrid numerical scheme. This scheme calculates conservative and non-conservative terms separately, utilizing both the particle method and the finite volume method, and demonstrates its effectiveness.

**Takanori Ide (Josai University)**

Reconstruction of the inclusions in electrical impedance tomography using enclosure neural networks for experimental data

Electrical impedance tomography is a non-destructive imaging method for medical imaging. The inverse problem of reconstructing internal electrical conductivity from boundary measurements is nonlinear and highly ill-posed problem. Currently, there has been growing interest in combining analytical methods and machine learning to solve inverse problems. In this talk, we propose a method for reconstruction of the inclusions from boundary measurements using combining the enclosure method and neural networks. We demonstrate its performance using experimental data.

**Kenta Kobayashi (Hitotsubashi University)**

Error analysis of Lagrange interpolation on triangles and tetrahedra

We have been studying error analysis of Lagrange interpolation on triangles and tetrahedra, and have succeeded in obtaining error evaluations applicable to triangles and tetrahedra of arbitrary shapes. In this presentation, we will present an overview of these results.

**Xuefeng Liu (Tokyo Woman's Christian University)**

Rigorous computation for eigenvalue problems and solution verification for the Navier–Stokes equations

Recently, rigorous computation has emerged as a powerful tool for providing explicit error estimates for numerical solutions to boundary value problems and eigenvalue problems of differential operators. In this talk, I will summarize the latest results on rigorous computation, with a focus on rigorous eigenvalue estimation, and describe the underlying concepts in the verification of solutions to the Navier-Stokes equations, including techniques for handling three-dimensional domains. Additionally, I will outline the objectives and current progress of an ongoing project aimed at verifying the existence of multiple solutions to the Navier-Stokes equations in 3D domains.

**Kazunori Matsui (Tokyo University of Marine Science and Technology)**

Numerical Method for an Elastoplastic Model with Time-Dependent Thresholds

We propose a numerical scheme using a projection-based approach for evolution equations including elastoplastic models where the yield surface depends on time.

**Takehiro Matsumoto (Tohoku University)**

Finite element approaches to the thermal convection in the eye

We simulated thermal convections of aqueous humor flow under the Boussinesq approximation by finite element method. We applied different types of boundary conditions on surfaces such as iris, lens, cornea, brim of cornea, and ciliary body. We succeeded in reproducing two types of convection cells and their transient dynamics depending on patient postures.

**Takayasu Matsuo (The University of Tokyo)**

A new convergence estimate of an energy-preserving scheme for the KdV equation

In this talk we consider the structure-preserving numerical scheme for the KdV equation proposed in Furihata (1999), in the very early phase of structure-preserving methods for partial differential equations. The scheme preserves the Hamiltonian and mass, and by numerical experiments it proved its robustness even for larger solutions. However, its theoretical analysis has been left open until quite recently. In this talk we explain the difficulty of the theoretical analysis, and a new way to successfully obtain a guarantee, which gives the missing theory after the quarter century since the scheme is proposed.

**Yuto Miyatake (The University of Osaka)**

Statistical modelling for quantifying discretization errors

The quantitative evaluation of errors introduced by discretizing differential equations is increasingly important in contexts such as data assimilation and inverse problems. In this talk, we focus on a posteriori estimation of such errors. Our approach treats the discretization errors as random variables and estimates them using available data. Specifically, we introduce a constrained Gaussian model for the discretization error and present inference methods and corresponding algorithms for estimating their key parameters. This framework aims to mitigate unnoticed overconfidence in inverse problems.

**Haruka Nakamura (The University of Tokyo)**

On the discretization and convergence rates of an iterative method for Mean Field Games

Mean Field Games (MFGs) are typically formulated as a coupled system consisting of a Fokker–Planck (FP) equation, which describes the time evolution of the density, and a Hamilton–Jacobi–Bellman (HJB) equation, which describes the optimal control of the FP equation. Recently, several iterative schemes for MFGs have been proposed, and their convergence has been analyzed. However, error estimates concerning both the discretization parameters and the number of iterations have not been well established so far. In this study, we propose a finite difference approximation of the iterative scheme based on Generalized Conditional Gradient (GCG) methods. Therein, we employ the Cole–Hopf transformation to address the non-linearity of the HJB equation. Finally, we establish convergence rates for the schemes concerning both the discretization parameters and the number of iterations, and verify their validity by numerical experiments.

**Yumiharu Nakano (Institute of Science Tokyo)**

Convergence of differentiable approximation schemes for fully nonlinear parabolic equations

We develop a convergence framework for a broad class of differentiable approximation schemes for fully nonlinear parabolic equations. The theory provides sufficient conditions for convergence to the unique viscosity solution without relying on monotonicity assumptions. While motivated by kernel-based collocation methods, our main focus is on the abstract convergence analysis rather than any specific scheme.

**Makoto Okumura (Konan University)**

Recent progress in the structure-preserving schemes for the two-dimensional Cahn-Hilliard models with dynamic boundary conditions

Recently, Cahn–Hilliard models with dynamic boundary conditions have been proposed by Goldstein et al. and Liu-Wu. These models have characteristic conservation laws, and we have designed structure-preserving schemes for these models that retain such conservation laws and the total energy dissipation law in a discrete setting. In this talk, we will introduce the numerical examples and talk about the solvability of the proposed schemes. If possible, we will also show results on structure-preserving schemes using nonlinear difference operators.

**Norikazu Saito (The University of Tokyo)**

Decoupling iterative numerical methods for mean field games

Mean field games (MFGs) are formulated as nonlinear coupled systems of partial differential equations, consisting of the Fokker–Planck equation, which governs the density distribution of agents, and the Hamilton–Jacobi–Bellman equation, which describes the temporal evolution of their control inputs. Such systems arise in a broad range of applications, including crowd dynamics, control of autonomous vehicle fleets, mathematical biology, engineering, and economics. Since MFGs are typically posed as space–time boundary value problems, numerical schemes designed for standard initial value problems cannot be directly applied. This motivates the development of new computational methods together with a rigorous mathematical foundation. In this talk, I present an implementation-friendly approach based on a generalized conditional gradient (GCG) method and discuss its convergence properties. In particular, I report recent results, obtained in collaboration with H. Nakamura, for MFGs with local coupling terms.

**Shun Sato (Tokyo Metropolitan University)**

Linearly implicit dissipative integrators inspired by proximal DC algorithms from optimization

Partial differential equations describing various phenomena often satisfy dissipation laws, which are important both physically and mathematically. Numerical methods that preserve these laws are called dissipative integrators. Among known approaches, convex splitting is a popular technique that decomposes the energy into the difference of two convex functions, treating one part implicitly and the other explicitly. Convex splitting schemes are solvable and preserve dissipation without step size restrictions, but they are generally fully implicit, leading to high computational cost.

Recently, the relationship between numerical analysis of differential equations and continuous optimization has attracted growing interest. In this context, convex splitting is related to DC (Difference of Convex) algorithms. Since DC algorithms also have computational cost issues, proximal DC algorithms have been proposed to improve efficiency. Inspired by this idea, we propose linearly implicit dissipative integrators. We illustrate the effectiveness of our approach by applying it to the Swift–Hohenberg equation, where the dissipation law allows for rigorous mathematical analysis.

This is a joint work with Daiki Iwade, Yuki Yonekura, and Takayasu Matsuo.

**Yoshiki Sugitani (Josai University)**

Numerical simulation and identification of time-varying parameters in the glucose–insulin metabolic system for ICU patients

Tight blood glucose control is effective in improving the outcomes of critically ill patients, and numerical simulations of the glucose–insulin metabolic system represent one potential approach. However, their accuracy largely depends on the proper identification of patient-specific and time-varying parameters such as insulin sensitivity. In this talk, we present a method that combines numerical simulation with machine learning to identify these parameters from observable ICU data, and demonstrate its application to actual clinical data for predicting future blood glucose levels.

**Hiroshi Suito (Tohoku University)**

On the thermal convection in intraocular aqueous humor flows

In this talk, flow simulations for aqueous humor in the eye are presented. Aqueous humor is a liquid that fills the anterior chamber between the cornea and the lens, the flow of which is strongly related to the cause of glaucoma. Different types of thermal convection are generated due to the temperature difference between the body and the outside temperatures, which requires challenging FEM simulations.

**Haruki Takemura (The University of Tokyo)**

Error estimates of semi-Lagrangian schemes using high-order interpolation for nonlinear advection–diffusion equations

We present error estimates of fully semi-Lagrangian schemes for the initial value problem for one-dimensional nonlinear advection–diffusion equations with periodic boundary conditions. These schemes employ high-order spline or Hermite interpolation for the spatial discretization. We refer to the method as the fully semi-Lagrangian scheme because the semi-Lagrangian techniques are applied to both advection and diffusion. For interpolation of degree $(2s-1)$, we establish $L^2$-error estimate of order $O(\Delta t + h^{2s}/\Delta t)$ and $H^s$-error estimate of order $O(\Delta t + h^s/\Delta t^{1/2})$, where $\Delta t$ denotes the time step and $h$ denotes the spatial mesh size.

**Yasutoshi Taniguchi (The University of Tokyo)**

Dirichlet Boundary Stabilization Methods for Space–Time Finite Element Analysis: An Application to the Wave Equation

In space–time finite element method, the weak formulation is constructed at each time step and solved sequentially in the temporal direction. Continuity between time steps is weakly enforced through the discontinuous Galerkin method. This study investigates the temporal distribution of the forcing term associated with the Dirichlet boundary, using the Dirichlet problem of the wave equation as a case study. It is demonstrated that, even in cases where this temporal distribution is expected to be inherently smooth, numerical oscillations may occur due to the discontinuity of the temporal basis functions of the primary variables, among other factors. Methods for mitigating this issue are then explored.

**Takuya Tsuchiya (The University of Osaka)**

Recent development of the finite element error analysis on anisotropic meshes

Meshes that do not satisfy the shape-regularity condition is called anisotropic. In this talk I will present recent development of the finite element error analysis on anisotropic meshes. We deal with error estimations of Lagrange, Crouzeix–Raviart, and Raviart–Thomas elements. ”

**Shinya Uchiumi (Hokkaido University)**

A pressure-stabilized projection Lagrange–Galerkin scheme for the transient Oseen problem

We propose and analyze a pressure-stabilized projection Lagrange–Galerkin scheme for the transient Oseen problem. The proposed scheme inherits the advantages from the projection Lagrange–Galerkin scheme: computational efficiency and essential unconditional stability. Here we also use the equal-order approximation for the velocity and pressure, and add a symmetric pressure stabilization term. This enriched pressure space enables us to obtain accurate solutions for small viscosity.

**Yuki Ueda (Hokkaido University)**

Numerical computations of split Bregman method for fourth-order total variation flow

We propose the split Bregman framework for the fourth-order total variation flow. The fourth-order total variation flow has been studied in the context of application to image processing and models of crystal growth. Under some appropriate assumptions, it can be regarded as $H^{-1}$ gradient flow of the total variation energy, and then the numerical computation can be reduced to "$L^1$-regularized" minimization problem. In this talk, we present the split Bregman framework for it and demonstrate numerical examples for fourth-order problems under periodic boundary conditions."

# Short Biographies

## Chinese side

(The list is arranged in alphabetical order by surname.)

## Japanese side

(The list is arranged in alphabetical order by surname.)

**Yuki Chiba (The University of Tokyo)** Yuki Chiba is currently a postdoctoral researcher at the Graduate School of Mathematical Sciences, University of Tokyo. He received his PhD in Mathematical Sciences from the University of Tokyo in 2020. His research focuses on the numerical analysis of PDEs such as the finite element method. Recently, He is interested in error analysis for problems on smooth domains.

**Daisuke Furihata (The University of Osaka)** Daisuke Furuta is a professor and director of the D3 Center at Osaka University in Japan. He received his Ph.D. in Engineering from the University of Tokyo in 1997. His main research field is structure-preserving numerical methods, primarily for partial differential equations, and he is currently interested in discretization methods for cases where the definition space of the solution is curved. He also sees potential in incorporating neural networks into structure-preserving numerical methods involving complex discretization.

**Takanori Ide (Josai University)** Takanori Ide is a Professor at the Department of Mathematics and Information Science , Faculty of Science, Josai University. He is a Fellow of the Japan Society for Industrial and Applied Mathematics. With over 20 years of experience in the automotive industry, he has been engaged in research and development using mathematics to solve real-world problems. His current research interests is inverse problems governed by partial differential equations using numerical analysis and machine learning.

**Kenta Kobayashi (Hitotsubashi University)** Kenta Kobayashi is a professor at Hitotsubashi University. His research interests are in the field of numerical analysis, especially numerical verification methods and error analysis of finite element methods.

**Xuefeng Liu (Tokyo Woman's Christian University)** Xuefeng Liu is a Professor at Tokyo Woman's Christian University. He earned his bachelor's degree from the University of Science  Technology of China (1998-2003) and obtained his Master's and PhD degrees at the University of Tokyo by 2009. His research focuses on numerical error analysis for differential equations, especially verified computation for eigenvalue problems, and developing computer-assisted mathematical proofs for the Navier-Stokes equations in 3D domains.

**Kazunori Matsui (Tokyo University of Marine Science and Technology)** Kazunori Matsui is currently an Assistant Professor at the Department of Logistics and Information Engineering, Tokyo University of Marine Science and Technology. He received his Ph.D. degree in Science from Kanazawa University in 2022. His research interests are in numerical analysis and mathematical analysis of partial differential equations. In particular, he is interested in fluid problems and elastoplastic problems. Recently, he has also been working on universal approximation properties of neural networks.

**Takehiro Matsumoto (Tohoku University)** Takehiro Matsumoto is currently a second-year master's course student at the Mathematical Institute, Graduate School of Science, Tohoku University. He originally worked in pure mathematics, but he later shifted his research focus to applied mathematics. He now concentrates on computational fluid dynamics and the application of causal inference to environmental problems.

**Takayasu Matsuo (The University of Tokyo)** Takayasu Matsuo is a professor at the Department of Mathematical Informatics, the Graduate School of Information Science and Technology, the University of Tokyo. He defended his Ph.D in 2003 at the University of Tokyo. His main research areas include numerical differential equations and numerical linear algebra. Recently, his interest has shifted into numerical analysis approaches for optimizations and deep neural networks.

**Yuto Miyatake (The University of Osaka)** Yuto Miyatake is currently an Associate Professor at D3 Center, the University of Osaka. He received his Ph.D. from the University of Tokyo in 2015. His research focusing on geometric integration of evolution equations — with or without machine learning techniques —, numerical linear algebra, and computational uncertainty quantification.

**Haruka Nakamura (The University of Tokyo)** Haruka Nakamura is a first-year Ph.D student at the graduate school of Mathematical Sciences, The University of Tokyo. His research focuses on the numerical analysis of Mean Field Game equations. In particular, he is interested in mass conservation in Fokker-Planck equations and in the construction of numerical schemes for Hamilton-Jacobi-Bellman equations, e.g. semi-Lagrangian approximations.

**Yumiharu Nakano (Institute of Science Tokyo)** Yumiharu Nakano is currently an Associate Professor at Department of Mathematical and Computing Science, Institute of Science Tokyo. He received his Ph.D degree in Mathematics from Hokkaido University in 2005. His research interests include stochastic differential equations, estimation and control of stochastic processes, and numerical analysis

**Makoto Okumura (Konan University)** Makoto Okumura is currently a Lecturer at the Faculty of Intelligence and Informatics, Konan University. He earned his Ph.D. in Information Science and Technology from Osaka University in 2021. His research focuses on numerical analysis of partial differential equations, especially structure-preserving methods. Recently, nonlinear difference operators have been proposed to bring out other superior features instead of relaxing linearity, and he has been working on designing numerical schemes using these differences.

**Norikazu Saito (The University of Tokyo)** Norikazu Saito is currently a Professor at the Graduate School of Mathematical Sciences and Director of the Interdisciplinary Center for Mathematical Sciences at The University of Tokyo. He received his Ph.D. from Meiji University in1999. He has served as a Director of the Japan Society for Industrial and AppliedMathematics and a Council Member of the Mathematical Society of Japan. His research focuses on the mathematical theory of the finite element method and finite volume method for nonlinear PDEs, such as the Keller-Segel equations and the Navier-Stokes equations. He is also interested in the mathematical foundations of the fictitious domain method and the immersed boundary method. He serves as an editor for international journals, including Numerische Mathematik and Journal of Scientific Computing.

**Shun Sato (Tokyo Metropolitan University)** Shun Sato is currently an Associate Professor in the Department of Mathematical Sciences, Graduate School of Science, Tokyo Metropolitan University. He received his Ph.D. from the University of Tokyo in 2019. His main research interests lie in geometric numerical integration for ordinary differential equations and evolutionary differential equations. Recently, his work has focused on exploring the connections between continuous optimization methods and numerical analysis, with the aim of developing more efficient algorithms for both fields.

**Yoshiki Sugitani (Josai University)** Yoshiki Sugitani is currently an Associate Professor in both the Department of Mathematics and Information Science and the Center for Mathematics and Data Sciences at Josai University. He received his Ph.D. in Mathematical Sciences from the University of Tokyo in 2017. His research interests include the application of numerical simulation and machine learning to clinical data analysis.

**Hiroshi Suito (Tohoku University)** Hiroshi Suito is a professor and principal investigator at the Advanced Institute for Materials Research (WPI-AIMR) and director of the Mathematical Science Center for Co-creative Society (MathCCS), Tohoku University, Japan. He obtained his PhD in 1998 from Chiba University after working at several IT companies. His main research fields are applied mathematics and numerical simulations addressing widely diverse problems. He was awarded the Commendation for Science and Technology by the Ministry of Education, Culture, Sports, Science and Technology (MEXT) in 2019.

**Haruki Takemura (The University of Tokyo)** Haruki Takemura is a second-year Ph.D. student at the Graduate School of Mathematical Sciences, The University of Tokyo. His research focuses on the mathematical analysis of semi-Lagrangian methods for fluid dynamics, in particular the cubic interpolated pseudo-particle (CIP) method, a semi-Lagrangian scheme involving the Hermite interpolation.

**Yasutoshi Taniguchi (The University of Tokyo)** Yasutoshi Taniguchi is currently a JSPS Postdoctoral Research Fellow at the Graduate School of Mathematical Sciences, The University of Tokyo. He received his Ph.D. in Mechanical Engineering from Waseda University in 2024. His research focuses on the space–time finite element method for continuum mechanics with isogeometric discretization. Recently, his research has focused on developing accurate methods for computing boundary forces in space–time finite element analysis.

**Takuya Tsuchiya (The University of Osaka)** Takuya TSUCHIYA retired Ehime University in March 2022, and became a professor emeritus. He is now a guest researcher of Osaka University.

**Shinya Uchiumi (Hokkaido University)** Shinya Uchiumi is currently a specially appointed associate professor at Hokkaido University. The main research area is numerical analysis on partial differential equations. Especially, he is working on the finite element methods for fluid problems, such as the Navier-Stokes equations. His research results are on the method of characteristics a.k.a. Lagrange-Galerkin method, and on higher order approximation of the pressure. He received his DSc degree from Waseda University in 2017. He worked in Waseda University from 2017 to 2018, and Gakushuin University from 2018 to 2023. After moving to Hokkaido University in 2023, he is working on a project whose purpose is to predict diabetes.

**Yuki Ueda (Hokkaido University)** Yuki Ueda is currently an Associate Professor in Research Institute for Electronic Science, Hokkaido University. He received PhD degree from the University of Tokyo in 2018. He got appointments as postdoctoral research fellow in the University of Tokyo, The Hong Kong Polytechnic University and Waseda University. His research area in numerical analysis of partial differential equations, and research interests include finite element method, isogeometric analysis, Navier-Stokes equation, total variation flow, and so on.